COMPARATIVE ANALYSIS OF FUZZY VALUE SIMILARITY MEASUREMENTS

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ABSTRACT

In this work, we looked into how entropy and similarity measurements relate to fuzzy sets. We noted that the fuzzy entropy is equal to the separation between the fuzzy set and the corresponding crisp set. We also use illustrative examples to demonstrate and discuss the properties of the fuzzy values and similarity measure. it is demonstrated that some features hold for some measures but not for others, and that some properties are shared by all measures. Last but not least, we developed a measure of similarity from and demonstrated using a straightforward example that the maximum similarity measure can be attained using a minimum entropy formulation.

Keywords: Fuzzy values , similarity measures, distance measure ,crisp set

INTRODUCTION:

The definitions of closeness measure and roughly equal fuzzy sets as well as the idea of approximating fuzzy values have all been introduced in [2]. a similarity index based on the greatest difference between equivalent membership grades, was implied, along with several hazy qualities sets related to this metric were displayed. They evaluated the similarity of fuzzy value measurements .The metrics looked at in [3] include:

(1) The measurement that is based on intersection and union procedures.
(2) The maximal difference-based measure.
(3) The measurement that takes into account membership grade disparities and the total.

It has been proven by earlier researchers [1],[2],[5],[6][10]and [11] that a fuzzy set's entropy is a measure of its fuzziness. Zadeh was the first to suggest fuzzy entropy as a fuzziness indicator; Pal and Pal studied classical Shannon information entropy; Kosko thought about the connection between distance measurement and fuzzy entropy; and Liu put forth an axiomatic definition of entropy, distance measures, and similarity measures and discussed the connections between these three ideas. This study is significant since it may offer us some helpful information for choosing an appropriate similarity metric in fuzzy set applications.

In this research, we expand the work of [8] to further explore fuzzy value similarity measures. the matching function S that we provided in [11]. The choice of the measures to be employed in fuzzy set applications may be influenced by the fact that it has been demonstrated that some qualities are shared by various measures and that others do not hold for all of them. The definitions of o composition and composition from [8] are briefly reviewed in the sections that follow.

SOME BASIC NOTATIONS AND DEFINITIONS :

The definitions of ∘ composition and α composition from [8] are briefly reviewed in the sections that follow. Let x and y be the scalar and let A be the fuzzy set of the universe of discourse U where U=$\sum_{i=1}^{m} u_i$ and let A'denote the complement of the fuzzy set A.

Let I, O and M denote the unit, Zero and 0.5 fuzzy sets, i.e. all membership grades in the fuzzy sets being equal to 1.0, 0 and 0.5, respectively, where the following notation will be used

$$x \cup y = \max(x, y)$$

$$x \cap y = min(x, y)$$

And

$$\text{I} ¿ \sum_{i=1}^{m} 1.0 \times u_i$$

$$\text{O} ¿ \sum_{i=1}^{m} 0 \times u_i$$

$$\text{M} ¿ \sum_{i=1}^{m} 0.5 \times u_i$$

The ∘ composition of the vector $a=(a_1, a_2, \ldots\ldots, a_m)$,corresponding to the fuzzy subset A of U, with the mat$rix\ R=[r_{ij}]$,corresponding to the fuzzy relation R of U×V, where U×V = $\sum_{i=1}^{m}\sum_{j=1}^{n} u_i v_j$ and is denoted by $a \circ R$ and is equal to the vector C$¿(c_1, c_2, \ldots\ldots, c_n)$, where

$$c_j = (a_i \cap r_{ij})$$

∩ denotes the minimum operator, and ∪ denotes the maximum operator.

The α composition of a scalar x with a scalar y which is denoted by is $x\alpha y$ is defined by

$$x\alpha y = \begin{cases} 1\, if\ x \le y \\ y\ otherwise \end{cases}$$

The α composition of the vector $s=(s_1, s_2, \ldots\ldots\ldots, s_m)$ with the scalar x is formed by substituting each element s_i of s with $s_i \alpha x$.the α composition of the matrix R with the vector $s=(s_1, s_2, \ldots\ldots\ldots, s_m)$ is formed by substituting each column vector r_j of R with $r_j \alpha s_j \wedge ¿$ is denoted by R α s. $(R\alpha s)$ and is denotes the vector whose elements are formed by taking the minimum element of the respective row vector of R α s.

MEASURE BASED ON INTERSECTION AND UNION OPERATIONS

The grade of similarity $M_{x,y}$ of the fuzzy sets x and y is defined by

$$M_{X,Y} ¿ \frac{\sum_i (x_i \cap y_i)}{\sum_i (x_i \cup y_i)} = \frac{|X \cap Y|}{|X \cup Y|}$$

If, given a small nonnegative number δ, X and Y are substantially equal (denoted by A- B), then

$$M_{X,Y} \le \delta$$

The distance between X and Y is said to be measured by the number δ.

Properties of $M_{X,Y}$,The following characteristics of $M_{X,Y}$ are accurate.

$(M_1)\ M_{X,Y} = M_{Y,X}$

$(M_2)\ X = Y \Longleftrightarrow M_{X,Y} = 1.$

$(M_3)\ X \cap Y = 0 \Longleftrightarrow M_{X,Y} = 0$

$(M_4)\ M_{X,X'} = 1 \Longleftrightarrow$ X=M.

¿¿)$M_{X,X'} = 0 \Longleftrightarrow$ X=I or X=0.

Properties of approximately equal fuzzy sets:-

$(M ¿¿ 6) ¿$ X ~ Y does not necessarily imply that

(X∪Z) ~(Y∪Z)

Consider x = $(0.3, 0.7, 0.5), y = (0.2, 0.4, 0.6)$ and z= $(0.1, 0.3, 0.8)$.it follows that

$$M_{X,Y} = \frac{0.2+0.4+0.5}{0.3+0.7+0.6} = \frac{1.1}{1.6} = 0.687$$

And

$$M_{X\cup Z,Y\cup Z}=\frac{(0.3+0.7+0.8)\cap(0.2+0.4+0.8)}{(0.3+0.7+0.8)\cup(0.2+0.4+0.8)}=\frac{0.2+0.4+0.8}{0.3+0.7+0.8}=\frac{1.4}{1.8}=0.777$$

i.e. $M_{X\cup Z,Y\cup Z}>M_{X,Y}$

which means that the proximity measure of X∪Z and Y∪Z is greater then that of X and Y.

thus X~Y does not necessarily imply that (X∪Z) ~(Y∪Z)

(M_7) X~Y does not necessarily imply that (X∩Z) ~(Y∩Z)

Zwick et al. [5] introduced a one parameter class of distance function defined as follows;

$$d_r(x,y)=\left[\sum_{i=1}^{m}|x_i-y_i|^r\right]^{\frac{1}{r}} \qquad (1)$$

Where x and y be an m-dimensional space, x = $(x_1,x_2,\ldots\ldots,x_m$¿ and y=$(y_1,y_2,\ldots\ldots,y_m$¿ ,when r=1,Eq. (1) becomes

$$d_1(x,y)=\sum_{i=1}^{m}|x_i-y_i| \qquad (2)$$

In the following ,we will investigate the properties of the measure $M_{X,Y}$ based on Eq. (2),where the grade of similarity $M_{X,Y}$ of the fuzzy sets X and Y is defined by

$$M_{X,Y}=1-\frac{\sum_{i=1}^{m}|x_i-y_i|}{n} \qquad (3)$$

If Consider x = (0.6,0.3,0.8) ,y =(0.5,1.0,0.7) and z =(0.6,0.3,0.5).it follows that

$$M_{X,Y}=¿\,1-\frac{0.1+0.7+0.1}{3}=0.167$$

And x ∩ y = $(0.5,0.3,0.7)$, y ∩ z = $(0.5,0.3,0.5)$

$$M_{X\cap Y,Y\cap Z}=1-\frac{0.0+0.0+0.2}{3}=0.934$$

i.e. $M_{X\cap Y,Y\cap Z}>M_{X,Y}$ which mean that the proximity measure of (X∩Z) and (Y∩Z) is greater than that of X and Y .

Thus X~Y does not necessarily imply that (X ∩Z) ~(Y∩Z).

(M_8) X~Y does not necessarily imply that $X\circ R\sim Y\circ R$.

Consider x = (0.6 ,0.5,0.6) ,y = (0.3 ,0.6,0.7)

And $R = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.5 \\ 0.3 & 0.6 \end{bmatrix}$

It follows that

$$M_{X,Y} = 1 - \frac{0.3+0.1+0.1}{3} = 1 - 0.433 = 0.567$$

$X \circ R = (0.47, 0.75)$, $Y \circ R = (0.39, 0.75)$

$$M_{X \circ R, Y \circ R} = 1 - \frac{(0.08+0.00)}{2} = 0.96$$

i.e. $M_{X \circ R, Y \circ R} > M_{X,Y}$,which means that the proximity measure of $X \circ R \wedge Y \circ R$ is greater than that of X and Y.

thus X~Y does not necessarily imply that $X \circ R \sim Y \circ R$.

(M_9) R~S does not necessarily imply that $X \circ R \sim X \circ S$ consider

$R = \begin{bmatrix} 0.2 & 0.6 & 0.7 \\ 1 & 0.2 & 0.5 \\ 0.5 & 0.3 & 1 \end{bmatrix}$, $S = \begin{bmatrix} 1 & 0.3 & 0.4 \\ 0.6 & 0.6 & 0.1 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$, x =(0.9,0.5,0.6)

It follows that

$$M_{R,S} = \frac{\left(1 - \frac{0.8+0.3+0.3}{3}\right) + \left(1 - \frac{0.4+0.4+0.4}{3}\right) + \left(1 - \frac{0.3+0.0+0.9}{3}\right)}{3}$$

$$= \frac{0.0+0.67+0.4}{3} = 0.40$$

$X \circ R$= (0.93 , 0.82 , 1.48) $X \circ S$ = (1.32 ,0.75 , o.47)

$$M_{X \circ R, X \circ S} = 1 - \frac{(0.39+0.07+1.01)}{3} = 1 - 0.49 = 0.51$$

i.e. $M_{X \circ R, X \circ S > M_{R,S}}$ which means that the proximity measure of $X \circ R \wedge X \circ S$ is greater than that of R and S.

thus R~S does not necessarily imply that $X \circ R \sim X \circ S$.

(M_{10}) α- composition

Let $\cap(R\alpha s) = f$ and $\cap(R\alpha t)$ =g

And let F and G be the fuzzy sets with membership vectors equal to f and g ,respectively.

S ~T does not necessarily imply that F~G. if consider s = (0.2,0.5,0.6) ,t =(0.5, 0.6,0.5)

R ¿ $\begin{bmatrix} 1 & 0.7 & 0.3 \\ 0.1 & 0.6 & 0.5 \\ 0.3 & 0.5 & 1 \end{bmatrix}$

It follows that

$M_{S,T} = 1 - \frac{0.3+0.1+0.1}{3}$ = 1- 0.16 = 0.84

R α s = $\begin{bmatrix} 0.2 & 0.5 & 1 \\ 1 & 0.5 & 1 \\ 0.2 & 0.5 & 0.6 \end{bmatrix}$, R α t¿ $\begin{bmatrix} 0.5 & 0.6 & 1 \\ 1 & 1 & 0.5 \\ 1 & 1 & 0.5 \end{bmatrix}$

f = ∩ (R α s)= (0.2 ,0.5,0.2)

g = ∩ (R α t) = (0.5 , 0.5, 0.5)

$M_{F,G}$= $1 - \frac{0.3+0.0+0.3}{3}$ = 1- 0.2 = 0.87

i.e. $M_{F,G} > M_{S,T}$,which means that the proximity measure of F and G is greater than that of S and T.

thus S ~ T does not necessarily imply that F ~ G.

(M_{11}) similarly ,it can be shown that if R and S are the matrices corresponding to the fuzzy relations R and S, if ∩(R α t) = f and ∩(S α t) = k, then

R ¿ $\begin{bmatrix} 1 & 0.1 & 0.3 \\ 0.7 & 0.2 & 0.8 \\ 0.3 & 0.5 & 1 \end{bmatrix}$, S ¿ $\begin{bmatrix} 1 & 0.5 & 0.9 \\ 0.5 & 0.8 & 0.7 \\ 0.4 & 0.7 & 0.2 \end{bmatrix}$,t= (0.5 ,0.8,0.3)

It follows that

$$M_{R,S} = \frac{\left(1-\frac{0.0+0.4+0.5}{3}\right)+\left(1-\frac{0.2+0.6+0.1}{3}\right)+\left(1-\frac{0.1+0.2+0.8}{3}\right)}{3} = \frac{0.7+0.7+0.64}{3} = 0.68$$

R α t = $\begin{bmatrix} 0.5 & 1 & 0.3 \\ 0.5 & 1 & 0.3 \\ 1 & 1 & 0.3 \end{bmatrix}$, $S \alpha t = \begin{bmatrix} 0.5 & 1 & 0.3 \\ 0.5 & 0.8 & 0.3 \\ 1 & 1 & 1 \end{bmatrix}$

∩(R α t) = (0.3,0.3,0.3) =f , ∩(S α t) = (0.3,0.3,1) = k

$M_{F,K}$ = 1- $\frac{0.0+0.0+0.7}{3}$ = 0.77

i.e. $M_{F,K} > M_{R,S}$,which means that the proximity measure of F and K is greater than of R and S. thus ,R~ S does not necessarily imply that F~ K.

DETERMINE THE LARGEST DIFFERENCE.

The fuzzy sets A and B's grade of similarity $W_{A,B}$ is defined by

$$W_{A,B} = 1\text{- } max_i(|a_i - b_i|)$$

For the $W_{A,B}$ proximity measure definitions are comparable to those of Section 3's definitions of approximately equal fuzzy sets.

$W_{A,B}$are has the qualities that are true:

$(W_1)\, W_{A,B} = W_{B,A}$

$(W_2)\, A = B \Longleftrightarrow W_{A,B} = 1.$

$(W_3)\, A \cap B = 0 \Longleftrightarrow W_{A,B} = 1\text{- } max_i(a_i - b_i)$

$(W_4)\, W_{A,A'} = 1 \Longleftrightarrow$ A=M.

¿¿)$W_{A,A'} = 0 \Longleftrightarrow$ A or A' are normal fuzzy sets.

$(W$¿¿$6)$¿A~B ⟹ (A∪C) ~(B∪C)

¿¿) A~B ⟹ (A∩C) ~(B∩C)

¿¿) A~B ⟹ (A∘R) ~(B∘R)

¿¿) R~S ⟹ (A∘R) ~(A∘S)

¿¿) X~Y does not necessarily imply that F~G.

¿¿) R~S does not necessarily imply that F~K

MEASURE USING AN ENTROPY-BASED CORRESPONDING CRISP SET

The derivations of similarity measures and their applications in the computation of the degree of similarity based on distance measures are the focus of all studies on similarity measures. the similarity measure has also been given an axiomatic definition by Liu [4].The four characteristics of the similarity measure ∀ A,B ∈ F(x) and ∀ D ∈ P(x) are as follows:

¿¿) s(A,B) = s(B,A) ∀ A,B ∈ F(x)

¿¿) s(D,D^c) = 0, ∀ D ∈ P(x)

¿¿) s$(C,C) = max_{A,B \in F}$ s(A,B) , ∀ C ∈ F(x)

¿¿) ∀ A,B,C ∈ F(x) ,if A ⊂ B ⊂ C ,then s$(A,B) \geq$ s(A,C) and s$(B,C) \geq$ s(A,C)

Where F(x) denotes a fuzzy set, and P(X) is a crisp set.

The equivalent crisp set must be taken into consideration, according to an examination of the entropy for the fuzzy set. The crisp set "near" the fuzzy set A is represented by A_{near}.When μ(x) ≥ 0.5, the value of 0.5 is one; otherwise, it is zero. A_{far} is the complement of A_{near},

i.e. $A_{near}^{C} = A_{far}$.

We suggested the fuzzy entropy of fuzzy set A with respect to A_{near} as follows [13] .

$$e(A, A_{near}) = d(A \cap A_{near}[1]_x) + d(A \cup A_{near}[0]_x) - 1 \qquad (1)$$

Theorem presents the suggested measure of similarity between A and A_{near}.We use a proof of this case to demonstrate the value of this measure.

Theorem:

The total information about fuzzy set A and the corresponding crisp set A_{near},

$$S(A, A_{near}) + e((A, A_{near}) = d(A \cap A_{near}[0]_x) + d(A \cup A_{near}[1]_x)$$
$$+ d(A \cap A_{near}[1]_x) + d(A \cup A_{near}[0]_x) - 1 \qquad (2)$$

Equals one .

Proof. Eq. (2) implies that the sum of the similarity measure and fuzzy entropy equals one ,which is the total area in (1).in Eq. (2)

$d(A \cap A_{near}[0]_x) + d(A \cap A_{near}[1]_x) = 1$,and

$d(A \cup A_{near}[0]_x) + d(A \cup A_{near}[1]_x) = 1.$

Hence, $S(A, A_{near}) + e((A, A_{near}) = 1+1-1 = 1$ is satisfied.

It is now obvious that the whole information about fuzzy set A includes comparisons to the matching crisp set in terms of similarity and entropy.

CONCLUSIONS

To make a comparison of measures of similarity of fuzzy values, Three fuzzy value similarity measures, one of which was introduced in this study, have definitions and properties that have been described and compared. It has been demonstrated that several of these characteristics apply to all measures. When compared to other attributes, they differ in a number of ways. t was investigated how to analyze similarity and entropy in fuzzy collections .In order to create fuzzy entropies for fuzzy sets, the crisp set "near" the fuzzy set was taken into account .the distance measure was also used to derive the similarity measure between the fuzzy set and the matching crisp set. Additionally, we have confirmed the fact that the sum of the fuzzy entropy and the similarity measure between the appropriate crisp set and the fuzzy set equals a constant number.

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